

Toy Example

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Numerical Example: Linear Transformations of Random Vectors

- Let

$$\mathbf{X} \in \mathbb{R}^2, \quad \mu = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

- Transformation matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

- Transformed mean:

$$\mathbf{A}\mu = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

- Transformed covariance:

$$\mathbf{A}\Sigma\mathbf{A}' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 2.5 \\ 2.5 & 1 \end{bmatrix}$$