Problem Set I

Gov 51, Spring 2024

February 8, 2024

For this part of the problem set, we will go through instrumental variables, its assumptions and Wald estimator. Make sure to cite your sources if you have gotten help for this question.

1 Assumptions

Can you please list the major assumptions for instrumental variables and interpret them in a few sentences?

- Exclusion restriction.
- No defiers.

2 Type of observations

Write down four types of observations in a IV study. And list their behavior implications on the potentail outcome: $Y(Z_i, T_i(Z_i))$. Make sure you explain briefly of your proof.

- Compliers: See page 11 of Lec 4 slides.
- Always / never takers: See page 11 of Lec 4 slides.

3 Wald Estimator

What is Encouragement term? What is Intention to Treat (ITT) term? What is Wald Estimator?

- See page 14 of Lec 4 slides.

4 Bonus question

Assume we only have compliers and never takers (one-sided compliance). Prove that LATE (Wald Estimatro) is equal to ATT:

$$E(Y_i(1) - Y_i(0)|T_i = 1)$$

To make things easier, we also assume instrumental variable is randomized, so that:

$$E(Y_i(T_i)|Z_i = 1) = E(Y_i(T_i)|Z_i = 0)$$

Under one sided compliance, we can simplify Wald Estimator as:

$$\frac{E(Y_i|Z_i = 1) - E(Y_i|Z_i = 0)}{E(T_i|Z_i = 1)}$$

This is true because $E(T_i|Z_i = 0)$ is zero for both compliers and never-takers.

Let's then take a closer look at the numerator.

$$\begin{aligned}
&E(Y_i|Z_i = 1) - E(Y_i|Z_i = 0) \\
&= E(Y_i|Z_i = 1) - \underbrace{E(Y_i(0)|Z_i = 0)}_{\text{one-sided compliance}} \\
&= E(Y_i(1)|Z_i = 1, T_i = 1)P(T_i = 1|Z_i = 1) + E(Y_i(0)|Z_i = 1, T_i = 0)P(T_i = 0|Z_i = 1) - E(Y_i(0)|Z_i = 0) \\
&= E(Y_i(1)|Z_i = 1, T_i = 1)P(T_i = 1|Z_i = 1) - E(Y_i(0)|Z_i = 1, T_i = 1)P(T_i = 1|Z_i = 1) \\
&\quad + \underbrace{E(Y_i(0)|Z_i = 1, T_i = 1)P(T_i = 1|Z_i = 1) + E(Y_i(0)|Z_i = 1, T_i = 0)P(T_i = 0|Z_i = 1)}_{E(Y_i(0)|Z_i=1)} - E(Y_i(0)|Z_i = 0) \\
&= E(Y_i(1)|Z_i = 1, T_i = 1)P(T_i = 1|Z_i = 1) - E(Y_i(0)|Z_i = 1, T_i = 1)P(T_i = 1|Z_i = 1) \\
&\quad + \underbrace{E(Y_i(0)|Z_i = 1) - E(Y_i(0)|Z_i = 0)}_{\text{=0 due to randomization}} \\
&= E(Y_i(1) - Y_i(0)|Z_i = 1, T_i = 1)P(T_i = 1|Z_i = 1) \\
&= E(Y_i(1) - Y_i(0)|T_i = 1)E(T_i|Z_i = 1) \quad \text{exclusion restriction}
\end{aligned}$$

Thus we have:

$$\frac{E(Y_i|Z_i = 1) - E(Y_i|Z_i = 0)}{E(T_i|Z_i = 1)} = E(Y_i(1) - Y_i(0)|T_i = 1)$$

An alternative way to prove it: link (by Prof. Blackwell)