Lecture 11: Uncertainty and Inference

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Feb. 27 2024

Logistics

- Final project group assignment done.
- March 5th midterm review!
- March 7th midterm - usual lecture time.
  - 50% Conceptual + 50% Coding.
  - Week 1 to Week 5.

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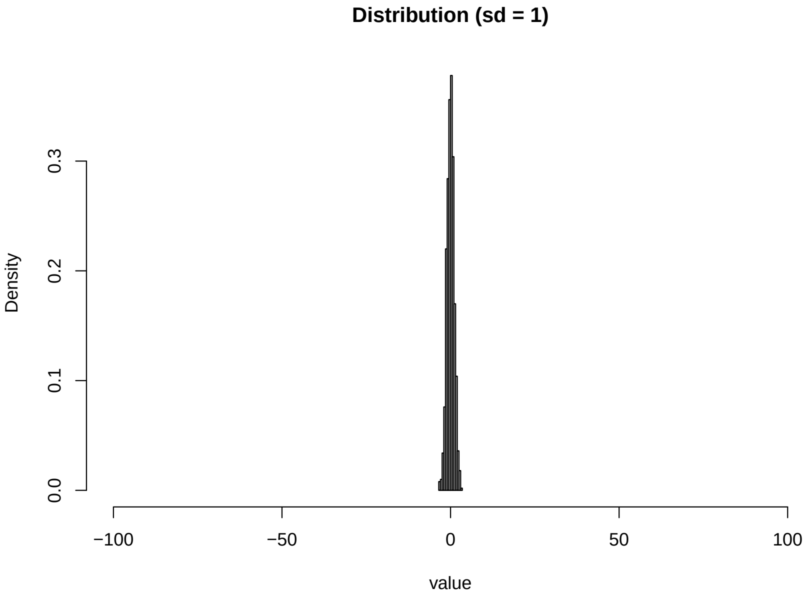
- Is $\hat{\beta}$ a good estimate of the true value? How certain are we ????

Standard Error as a Measurement for Uncertainty



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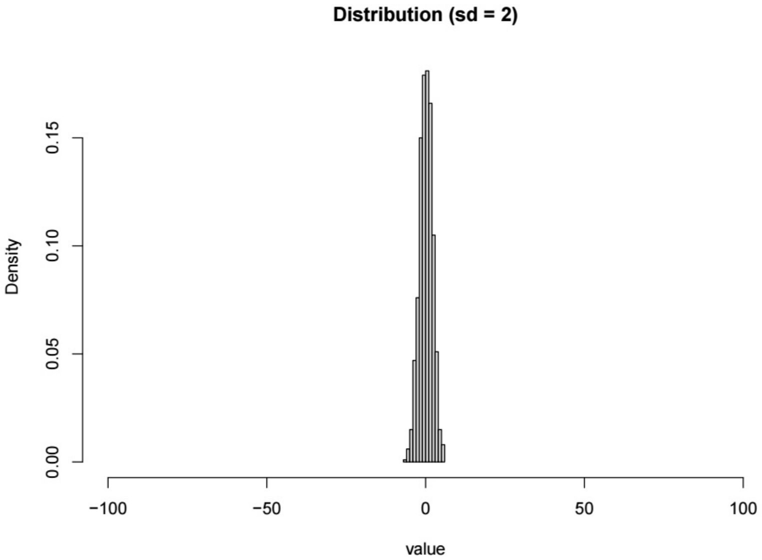

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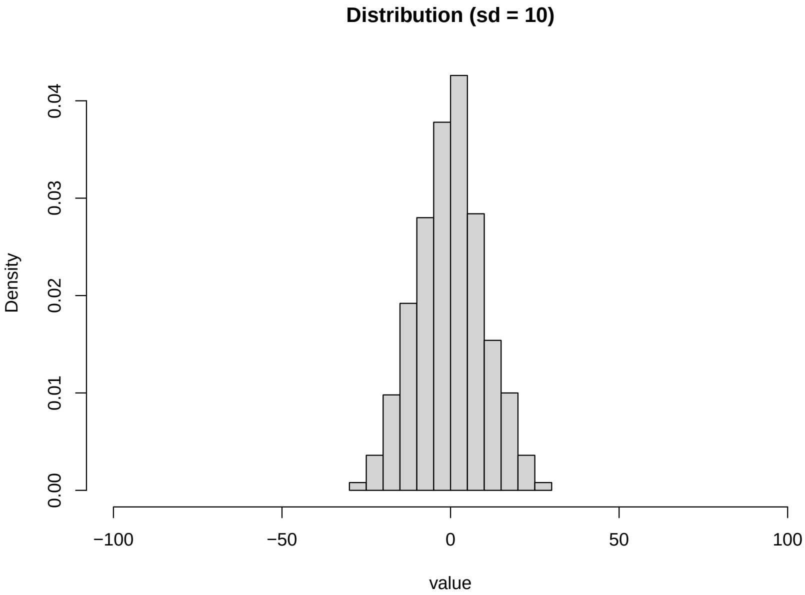

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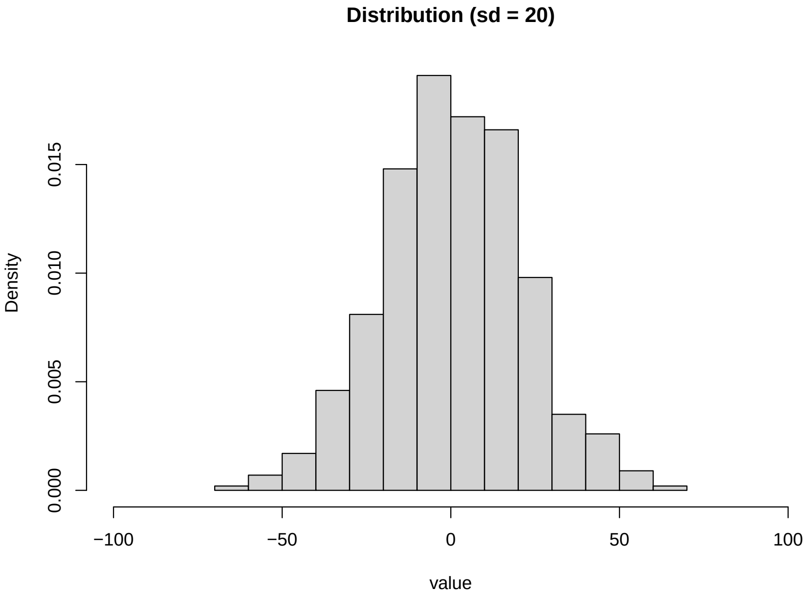

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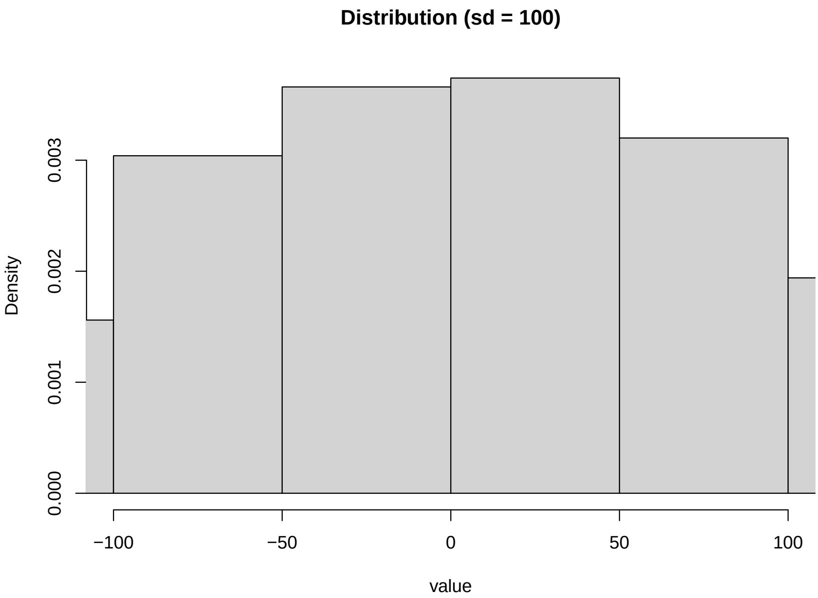

Standard Error as a Measurement for Uncertainty

- Standard error tells us how "spread out" our data is.
- It's not good or bad to have a high standard error.
  - In some scenario, we want to a **precise** estimate.
  - In other scenarios, we want to observe heterogeneity.

Variance and Bias

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- There are two ways to collect a sample:
  - We look for 1000 people who are Asian, female, 25 years old, no known illness, exercise twice per week, eat salad every day.

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    We will get a precise $\hat{\beta}$, but probably biased from the true value!
  - We look for 1000 people who are across races, genders, age groups and health conditions.

    We will get a noisy $\hat{\beta}$, but probably closer to the true value!

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$$
\begin{aligned}
\text{MSE} &= \mathbb{E}\left((\hat{\beta} - \beta)^2\right) \\
&= \mathbb{E}\left((\underbrace{\hat{\beta} - \mathbb{E}(\hat{\beta})}_{A} + \underbrace{\mathbb{E}(\hat{\beta}) - \beta}_{B})^2\right) \\
&= \mathbb{E}\left(A^2 + B^2 + 2AB\right) \\
&= \underbrace{\mathbb{E}\left((\hat{\beta} - \mathbb{E}(\hat{\beta}))^2\right)}_{\text{variance}} + \underbrace{\mathbb{E}\left(\mathbb{E}(\hat{\beta}) - \beta\right)^2}_{\text{bias}^2} + 2\mathbb{E}\left((\hat{\beta} - \mathbb{E}(\hat{\beta}))(\mathbb{E}(\hat{\beta}) - \beta)\right)
\end{aligned}
$$

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$$
\begin{aligned}
&\mathbb{E}\left((\hat{\beta} - \mathbb{E}(\hat{\beta}))(\mathbb{E}(\hat{\beta}) - \beta)\right) \\
&= \mathbb{E}\left(\hat{\beta}\mathbb{E}(\hat{\beta}) - \mathbb{E}(\hat{\beta})\mathbb{E}(\hat{\beta}) - \hat{\beta}\beta + \mathbb{E}(\hat{\beta})\beta\right) \\
&= \mathbb{E}(\hat{\beta})\mathbb{E}(\hat{\beta}) - \mathbb{E}(\hat{\beta})\mathbb{E}(\hat{\beta}) - \beta\mathbb{E}(\hat{\beta}) + \beta\mathbb{E}(\hat{\beta}) \\
&= 0
\end{aligned}
$$

Variance and Bias

$$\text{MSE} = \underbrace{\mathbb{E}\left((\hat{\beta} - \mathbb{E}(\hat{\beta}))^2\right)}_{\text{variance}} + \underbrace{\mathbb{E}\left(\mathbb{E}(\hat{\beta}) - \beta)\right)^2}_{\text{bias}^2}$$

$$= \underbrace{\mathbb{E}\left((\hat{\beta} - \mathbb{E}(\hat{\beta}))^2\right)}_{\text{variance}} + \underbrace{\left(\mathbb{E}(\hat{\beta}) - \beta)\right)^2}_{\text{bias}^2}$$

Because $\mathbb{E}(\hat{\beta}) - \beta)$ is a constant!

Variance and Bias

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  - We can use the representative sample to reduce bias, at the cost of high variance.
- We will see OLS estimator is unbaised, with a closed form variance. (later)

OLS Coefficients

$$Y = \beta X + \beta_0 + \epsilon$$

- Null Hypothesis:

$$\beta = 0$$

- P-value in a t-test:

$$\frac{\hat{\beta} - 0}{\text{std error}\hat{\beta}}$$

P-value$= 0.001$ **indicates that if null hypothesis were true, we would get this value of** $\hat{\beta}$ **with a probability of 0.001.**

- Confidence: Thus, we can reject the null with a confidence of 0.999.

Type of errors

	Reject H_0	Accept H_0
H_0 is true		Correct
H_0 is false	Correct	

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- Type I error: Reject the null hypothesis when it is true.

  False negatives

  - This is fine when we want the test to be aggressive.

Type of errors

	Reject H_0	Accept H_0
H_0 is true	**Type I error**	Correct
H_0 is false	Correct	**Type II error**

- Type I error: Reject the null hypothesis when it is true.

  False negatives

  - This is fine when we want the test to be aggressive.
- Type II error: Accept the null hypothesis when it is false.

  False positives

  - This is fine when we want the test to be conservative.

Type or errors

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- Type I error: A healthy person diagnosed with Diabetes.
- Type II error: A Diabetes patient diagnosed as healthy.

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- Do elected officials update their policy positions in response to expert evidence? (Lee, 2021, APSR)

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Hypotheses

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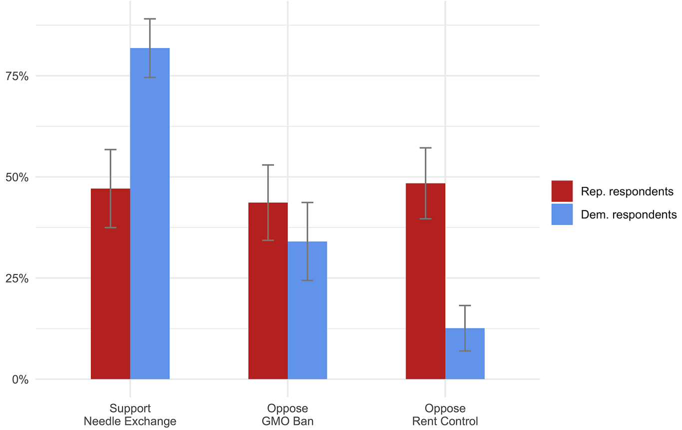
  Policy makers will be less likely update their beliefs or preferences in the direction of the evidence when the evidence is uncongenial.
- Accuracy Motivation Hypothesis
  Policy makers will update their beliefs or preferences in the direction of the expert evidence irrespective of the congeniality of the evidence.

Issues

- Needle exchange, GMOs, and rent control.



Hypotheses Test

- Support on needle exchange program

$$Y_i = \beta_0 + \beta_1 \cdot \text{Republican} + \beta_2 \cdot \text{new info} + \beta_3 \cdot \text{Republican} \cdot \text{new info}$$

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$$\beta_2 > 0$$

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- Accuracy motivated reasoning:

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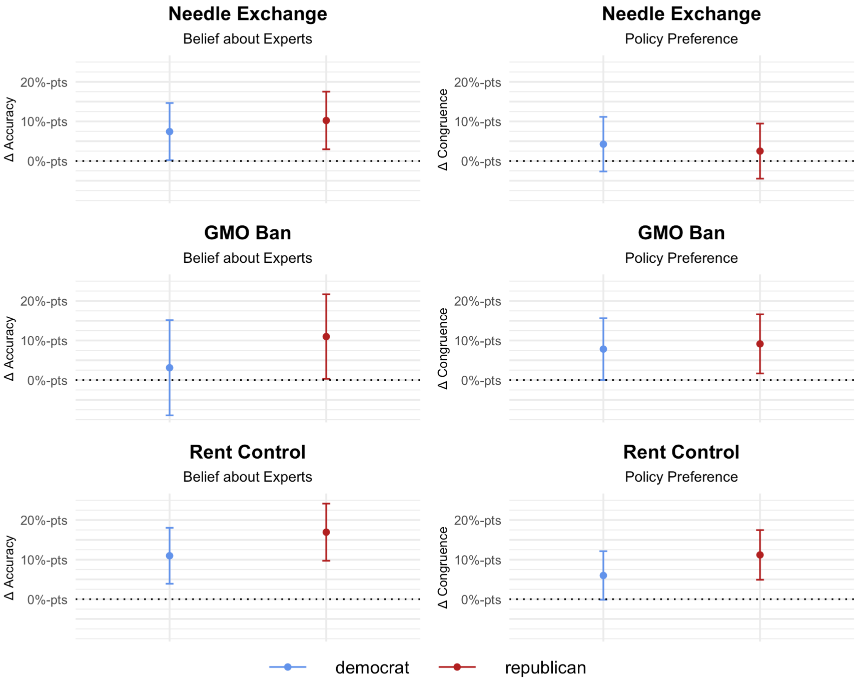
- Directional motivated reasoning:

  $$\beta_3 > 0$$

  Conditional on given new info, partisanship will still affect support.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Belief about Experts
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Policy Preference
Δ Accuracy
Δ Congruence
20%-pts
10%-pts
0%-pts
democrat
republican

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- We could be wrong in two ways: type I or II error.
- Next lecture: we will take a closer look at uncertainty for OLS coefficients.