Lecture 12: Uncertainty and Inference

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Midterm Week Plan

- Tuesday lecture: Review.
- Thursday lecture: Exam.
- Conceptual questions: Closed-book test.
- Coding questions: Semi closed-book
  - Embedded manual of `R-studio`: ?function-name
  - Previous coding tasks and pset.

Uncertainty for the OLS coefficient

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- First, we want to show $\hat{\beta}$ is unbiased for true β.
- Then, we will take a look at **how certain** we can be for it.

Unbiasedness

$$\begin{aligned}
\hat{\beta} &= \frac{\sum X_i Y_i}{\sum X_i^2} \\
&= \frac{\sum X_i(\beta X_i + \epsilon_i)}{\sum X_i^2} \\
&= \frac{\beta \sum X_i^2 + \sum X_i \epsilon_i}{\sum X_i^2} \\
&= \beta + \frac{\sum X_i \epsilon_i}{\sum X_i^2}
\end{aligned}$$

So we have:

$$\hat{\beta} - \beta = \frac{\sum X_i \epsilon_i}{\sum X_i^2}$$

Unbiasedness

$$\begin{aligned}
\mathbb{E}(\hat{\beta} - \beta | X) &= \mathbb{E}\left(\frac{\sum X_i \epsilon_i}{\sum X_i^2} | X\right) \\
&= \frac{1}{\sum X_i^2} \sum \left(\mathbb{E}(\epsilon_i | X) X_i\right) \\
&= 0
\end{aligned}$$

In expectation, OLS can give us an unbiased estimate of the coefficient.

Homoskedasticity

- Error not dependent on covariates.

$$V(\epsilon_i|X) = V(\epsilon_i)$$

- ϵ_i and ϵ_j are independent, conditional on X

Variance of OLS coefficient

$$
\begin{aligned}
V(\hat{\beta}|X) &= V\left(\frac{\sum X_i Y_i}{\sum X_i^2}|X\right) \\
&= V\left(\frac{\sum X_i(\beta X_i + \epsilon_i)}{\sum X_i^2}|X\right) && \text{substitute in for } Y_i \\
&= V\left(\frac{\beta \sum X_i^2}{\sum X_i^2}|X\right) + V\left(\frac{\sum X_i \epsilon_i}{\sum X_i^2}|X\right) && \beta \text{ is a constant} \\
&= V(\beta|X) + \frac{1}{(\sum X_i^2)^2} V\left(\sum \epsilon_i X_i|X\right) \\
&= 0 + \frac{B}{A} V(\epsilon_i)
\end{aligned}
$$

homoskedastic error assumption

Brainstorm question

Please show

$$\hat{\beta} = \frac{\sum_{i=1}^{N} X_i(Y_i - \bar{Y})}{\sum_{i=1}^{N} X_i(X_i - \bar{X})} = \frac{\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{N}(X_i - \bar{X})^2}$$

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Furthermore, can you show the unbiased-ness using this form of $\hat{\beta}$?