Lecture 2: Reviews on DID

Naijia Liu

Jan 25, 2024

Figure: Please fill out our section survey, if you are under the "TBA" section, or wish to switch sections.

Difference in Difference

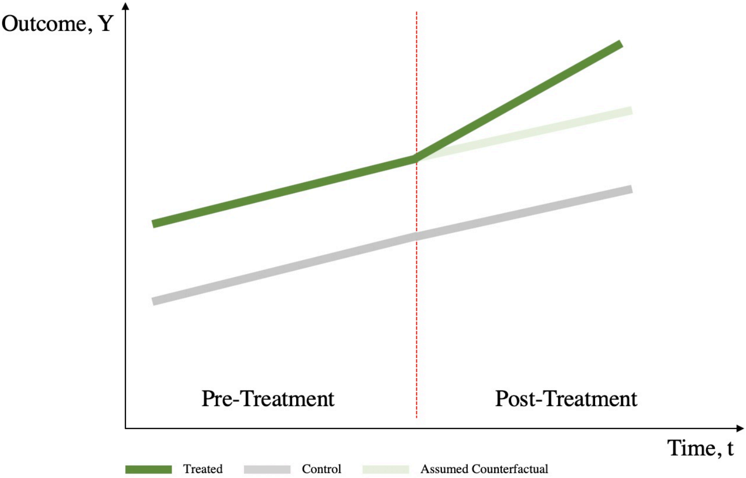
- Before and after difference of **control group** to infer what would have happened to **treatment group** without treatment.

Difference in Difference

- Before and after difference of **control group** to infer what would have happened to **treatment group** without treatment.
- We want to estimate:

$$\underbrace{\left(\overline{Y}^{\text{after}}_{\text{treated}} - \overline{Y}^{\text{before}}_{\text{treated}}\right)}_{\text{trend in treated group}} - \underbrace{\left(\overline{Y}^{\text{after}}_{\text{control}} - \overline{Y}^{\text{before}}_{\text{control}}\right)}_{\text{trend in control group}}$$

Parallel Trend



Parallel Trend

Assumption

$$\mathbb{E}\left(Y_{i1}(0) - Y_{i0}(1) | T_i = 1 \right) = \mathbb{E}\left(Y_{i1}(0) - Y_{i0}(0) | T_i = 0 \right)$$

treated and control group will share same difference, if the treated were not to be treated at $t = 1$.

Parallel Trend

Assumption

$$\mathbb{E}\left(Y_{i1}(0) - Y_{i0}(1) \middle| T_i = 1\right) = \mathbb{E}\left(Y_{i1}(0) - Y_{i0}(0) \middle| T_i = 0\right)$$

treated and control group will share same difference, if the treated were not to be treated at $t = 1$.

We are interested in:

$$\mathbb{E}\left(Y_{i1}(1) - Y_{i1}(0) \middle| T_i = 1\right)$$

For the treated at $t = 1$, what if they were not treated?

Proof

$$\mathbb{E}\left(Y_{i1}(0) - Y_{i0}(1) | T_i = 1 \right) = \mathbb{E}\left(Y_{i1}(0) - Y_{i0}(0) | T_i = 0 \right); \quad (Assn)$$

Proof

$$\mathbb{E}\left(Y_{i1}(0) - Y_{i0}(1) \middle| T_i = 1 \right) = \mathbb{E}\left(Y_{i1}(0) - Y_{i0}(0) \middle| T_i = 0 \right); \quad (Assn)$$
$$\mathbb{E}\left(Y_{i0}(1) - Y_{i1}(0) \middle| T_i = 1 \right) = \mathbb{E}\left(Y_{i0}(0) - Y_{i1}(0) \middle| T_i = 0 \right); \quad (\times - 1)$$

Proof

$$\mathbb{E}\left(Y_{i1}(0) - Y_{i0}(1) \middle| T_i = 1\right) = \mathbb{E}\left(Y_{i1}(0) - Y_{i0}(0) \middle| T_i = 0\right); \quad (Assn)$$

$$\mathbb{E}\left(Y_{i0}(1) - Y_{i1}(0) \middle| T_i = 1\right) = \mathbb{E}\left(Y_{i0}(0) - Y_{i1}(0) \middle| T_i = 0\right); \quad (\times - 1)$$

$$\mathbb{E}\left(-Y_{i1}(0) \middle| T_i = 1\right) = \mathbb{E}\left(-Y_{i0}(1) \middle| T_i = 1\right) + \mathbb{E}\left(Y_{i0}(0) - Y_{i1}(0) \middle| T_i = 0\right);$$

$(move\ Y_{i0}(1))$

Proof

$$\mathbb{E}\left(Y_{i1}(0) - Y_{i0}(1) | T_i = 1\right) = \mathbb{E}\left(Y_{i1}(0) - Y_{i0}(0) | T_i = 0\right); \quad (Assn)$$

$$\mathbb{E}\left(Y_{i0}(1) - Y_{i1}(0) | T_i = 1\right) = \mathbb{E}\left(Y_{i0}(0) - Y_{i1}(0) | T_i = 0\right); \quad (\times - 1)$$

$$\mathbb{E}\left(-Y_{i1}(0) | T_i = 1\right) = \mathbb{E}\left(-Y_{i0}(1) | T_i = 1\right) + \mathbb{E}\left(Y_{i0}(0) - Y_{i1}(0) | T_i = 0\right);$$

$$(move\ Y_{i0}(1))$$

$$\mathbb{E}\left(Y_{i1}(1) - Y_{i1}(0) | T_i = 1\right) = \mathbb{E}\left(Y_{i1}(1) - Y_{i0}(1) | T_i = 1\right) + \mathbb{E}\left(Y_{i0}(0) - Y_{i1}(0) | T_i = 0\right);$$

$$(add\ Y_{i1}(1))$$

Proof

$$\mathbb{E}\left(Y_{i1}(0) - Y_{i0}(1) \middle| T_i = 1\right) = \mathbb{E}\left(Y_{i1}(0) - Y_{i0}(0) \middle| T_i = 0\right); \quad (Assn)$$

$$\mathbb{E}\left(Y_{i0}(1) - Y_{i1}(0) \middle| T_i = 1\right) = \mathbb{E}\left(Y_{i0}(0) - Y_{i1}(0) \middle| T_i = 0\right); \quad (\times -1)$$

$$\mathbb{E}\left(-Y_{i1}(0) \middle| T_i = 1\right) = \mathbb{E}\left(-Y_{i0}(1) \middle| T_i = 1\right) + \mathbb{E}\left(Y_{i0}(0) - Y_{i1}(0) \middle| T_i = 0\right);$$

$$(move\ Y_{i0}(1))$$

$$\mathbb{E}\left(Y_{i1}(1) - Y_{i1}(0) \middle| T_i = 1\right) = \mathbb{E}\left(Y_{i1}(1) - Y_{i0}(1) \middle| T_i = 1\right) + \mathbb{E}\left(Y_{i0}(0) - Y_{i1}(0) \middle| T_i = 0\right);$$

$$(add\ Y_{i1}(1))$$

$$\mathbb{E}\left(Y_{i1}(1) - Y_{i1}(0) \middle| T_i = 1\right) =$$

$$\underbrace{\mathbb{E}\left(Y_{i1}(1) - Y_{i0}(1) \middle| T_i = 1\right) - \mathbb{E}\left(Y_{i1}(0) - Y_{i0}(0) \middle| T_i = 0\right);}_{\text{Difference in Difference estimator that we can obtain from data}}$$