GOV 51 Section

Week 2: DiD Review, Instrumental Variables Design

Pranav Moudgalya

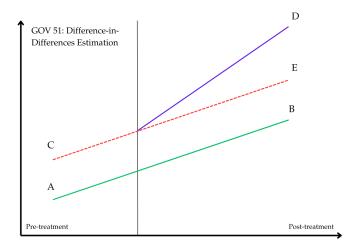
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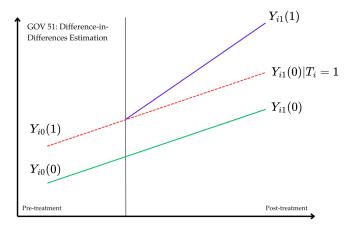
Review: Difference-in-Differences

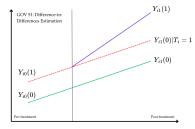
What is DiD?: A difference-in-differences design compares changes over time between two groups—one affected by a treatment and one not—to measure the impact of that treatment The main assumption is that, in the absence of the treatment, both groups would have followed parallel trends over time.

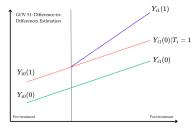
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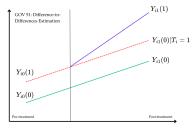






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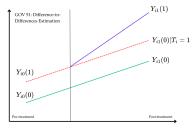


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Estimator:

$$\widehat{DiD} = \mathbb{E}[Y_{i1}(1) - Y_{i0}(1)|T_i = 1] - \mathbb{E}[Y_{i1}(0) - Y_{i0}(0)|T_i = 0]$$

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We have some other assumptions we have to make. That is, for an instrumental variable to be identified, it must:

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- ► IF we are able to meet these assumptions, we get a consistent estimate of the Local Average Treatment Effect (LATE)
 - Check-in: why is this only a local average treatment effect? What does "local" mean (hint: what group are we estimating a treatment effect for?)

True or false (discuss with a neighbor)?

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- Example 1: Researchers used state-level cigarette tax rates as an instrument to estimate the impact of maternal smoking on infant birth weight. Higher taxes reduce smoking rates by increasing costs, providing exogenous variation in smoking behavior. The exclusion restriction is plausible because cigarette taxes should only affect birth weight through smoking, not through any direct impact on fetal development. The intent-to-treat (ITT) effect is measured by comparing birth weights between high- and low-tax states, confirming that smoking behavior responds to the instrument.

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Example 2: The Vietnam War draft lottery provided an exogenous assignment of military service, allowing researchers to estimate its impact on lifetime earnings. Since draft numbers were randomly assigned, the instrument satisfies the exclusion restriction, as lottery status should affect earnings only through the likelihood of military service. The ITT effect was established by comparing the earnings of individuals with low versus high draft numbers, showing that the lottery assignment significantly influenced military participation.

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- Gym proximity might correlate with neighborhoods that attract health-conscious individuals who would exercise regardless of gym membership.

IV Notation



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Instrument: Z_i

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IV Observations

Discuss with a neighbor what this table illustrates.

IV: Encouragement & ITT

Turn to a neighbor: what does *encouragement* mean, and what does *ITT* refer to?

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Encouragement

$$\frac{\sum_{i=1}^{N} T_i Z_i}{\sum_{i=1}^{N} Z_i} - \frac{\sum_{i=1}^{N} T_i (1 - Z_i)}{\sum_{i=1}^{N} (1 - Z_i)}$$

$$\mathbb{E}(T_i \mid Z_i = 1) - \mathbb{E}(T_i \mid Z_i = 0)$$

This formula compares the fraction of people who actually took the treatment $(T_i = 1)$ in two groups:

- 1) Those who were encouraged to take it $(Z_i = 1)$
- 2) Those who were not encouraged $(Z_i = 0)$

Encouragement measures how much more likely people are to take the treatment if they were encouraged.

IV: Encouragement & ITT Intent-to-Treat (ITT)

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$$\frac{\sum_{i=1}^{N} Y_i Z_i}{\sum_{i=1}^{N} Z_i} - \frac{\sum_{i=1}^{N} Y_i (1 - Z_i)}{\sum_{i=1}^{N} (1 - Z_i)}$$

$$\mathbb{E}\left(Y_i(Z_i=1)\right) - \mathbb{E}\left(Y_i(Z_i=0)\right)$$

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$$\mathbb{E}\left(Y_i(Z_i=1)\right) - \mathbb{E}\left(Y_i(Z_i=0)\right)$$

This formula compares the average outcome (Y_i) between two groups based on an instrumental variable (Z_i) :

- ▶ People who were assigned $Z_i = 1$ (e.g., those who received some kind of encouragement or assignment to treatment).
- People who were assigned Z_i = 0 (e.g., those who did not receive encouragement or assignment).

It calculates the difference in average outcomes between these two groups (how much the outcome changes, on average, between the two levels of the instrumental variable)

IV: Wald Estimator



$$\frac{\mathbb{E}\left(Y_i(Z_i=1)\right) - \mathbb{E}\left(Y_i(Z_i=0)\right)}{\mathbb{E}\left(T_i(Z_i=1)\right) - \mathbb{E}\left(T_i(Z_i=0)\right)}$$

Average treatment effect among the compliers

By dividing ITT by encouragement, we isolate the **causal effect of treatment** for the group that actually complies with their assignment.

IV: Wald Estimator (by hand, and in R)

Fake Dataset for Wald Estimator Calculation

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Fake Dataset for Wald Estimator Calculation

ID	Ζ	Т	Y
1	1	1	9
2	1	1	8
3	1	0	6
4	1	1	10
5	0	0	7
6	0	0	5
7	0	1	9
8	0	0	6

Can you calculate the LATE using the Wald estimator?

IV: Wald Estimator (by hand)

The Wald estimator is given by:

$$\widehat{\mathsf{Wald}} = \frac{\mathbb{E}[Y|Z=1] - \mathbb{E}[Y|Z=0]}{\mathbb{E}[T|Z=1] - \mathbb{E}[T|Z=0]}$$
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Step 1: Compute $\mathbb{E}[Y|Z=1]$

$$\mathbb{E}[Y|Z=1] = \frac{9+8+6+10}{4} = \frac{33}{4} = 8.25$$

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Step 2: Compute $\mathbb{E}[Y|Z=0]$ $\mathbb{E}[Y|Z=0] = \frac{7+5+9+6}{4} = \frac{27}{4} = 6.75$ Wald Estimator (by hand) Step 3: Compute $\mathbb{E}[T|Z = 1]$

$$\mathbb{E}[T|Z=1] = \frac{1+1+0+1}{4} = \frac{3}{4} = 0.75$$

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Step 5: Compute LATE using the Wald estimator

$$\widehat{\mathsf{Wald}} = \frac{8.25 - 6.75}{0.75 - 0.25} = \frac{1.5}{0.5} = 3$$

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Thus, the LATE is 3!.

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setwd("/Users/pmoudgalya/Desktop/gov51")

- ► OR click Session → Set Working Directory → Choose Directory
- R Projects set your working directory to the folder that it is in

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 - So in some sense, we are language agnostic (you can use Python - but we can't help you with coding and won't go through the extra work of reviewing your code in case partial credit could be assigned)